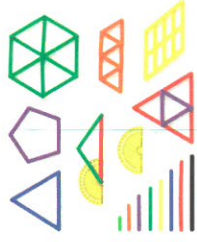
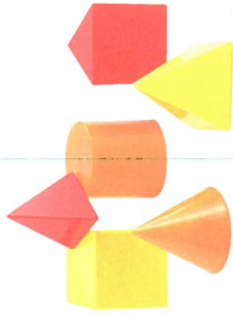


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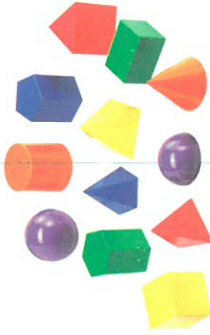
21362C **GeoStix™**



21314 Geometric Solids (6 pcs)



21317 Geometric Solids (12 pcs)



Written by Dr Paul Swan

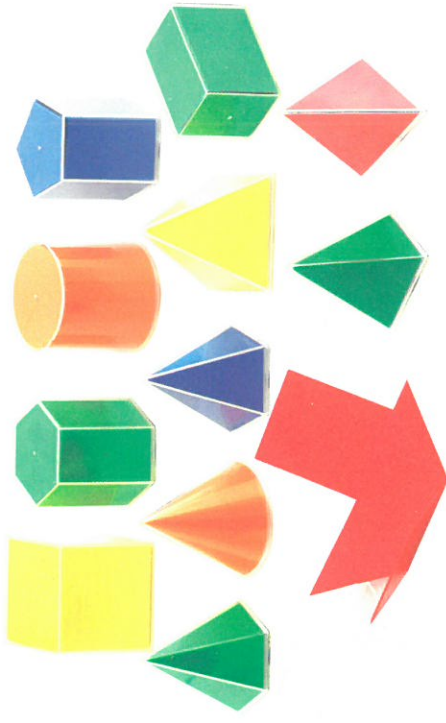
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# Folding 2D3D Geometric Solids

## Activity Guide



Use the Folding 2D3D Geometric Solids Kit to explore and study:

- Properties of 3D solids
- Relationships between 2D shapes and 3D solids
- Volume and capacity
- Surface area
- Nets

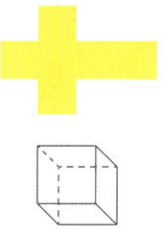


**WARNING: CHOKING HAZARD**  
Small parts. Not for children under 3 yrs.

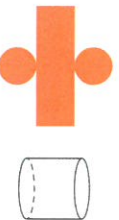
## 2D Shapes and 3D Solids



Square Pyramid



Square Prism (cube)



Cylinder



Triangular Pyramid



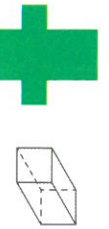
Triangular Prism



Cone



Rectangular Pyramid



Rectangular Prism



Pentagonal Pyramid



Pentagonal Prism

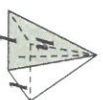


Hexagonal Pyramid



Hexagonal Prism

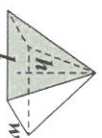
## Formulas - Pyramids



Square pyramid

$$\begin{aligned} \text{Volume} &= \text{length} \times \text{width} \times \text{height} \\ V &= 1/3 \times l \times l \times h \\ &= 1/3 l^2 h \end{aligned}$$

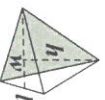
Surface area = sum of areas of each side



Rectangular pyramid

$$\begin{aligned} \text{Volume} &= 1/3 \times \text{area of base} \times \text{height} \\ V &= 1/3 \times l \times w \times h \\ &= 1/3 lwh \end{aligned}$$

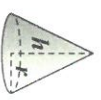
Surface area = sum of areas of each side



Triangular pyramid

$$\begin{aligned} \text{Volume} &= 1/3 \times \text{area of base} \times \text{height} \\ V &= 1/3 \times 1/2 l \times w \times h \\ &= 1/6 lwh \end{aligned}$$

Surface area = sum of areas of each side



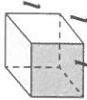
Cone (circular pyramid)

$$\begin{aligned} \text{Volume} &= 1/3 \text{ area of base} \times \text{height} \\ V &= 1/3 \times \pi r^2 \times h \\ &= 1/3 \pi r^2 h \end{aligned}$$

All pyramids

Volume = 1/3 the volume of a prism with the same base and height  
SA = sum of areas of each side

## Formulae - Prisms



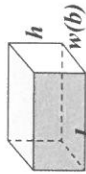
Cube

Volume = length x width x height

$$V = l \times l \times l = l^3$$

Surface area = area of one side x 6

$$SA = l \times l \times 6 = 6l^2$$



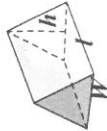
Rectangular prism

Volume = length x width x height

$$V = l \times w \times h = lwh$$

Surface area = sum of areas of each side

$$SA = w \times h \times 2 + w \times l \times 2 + h \times l \times 2 \\ = 2wh + 2wl + 2hl$$



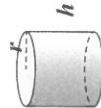
Triangular prism  
(equilateral)

Volume = area of end region x length

$$V = 1/2 \times \text{base} \times \text{height} \times \text{length} \\ = 1/2 \times w \times h \times l = 1/2 whl$$

Surface area = sum of areas of each side

$$SA = 1/2 wh \times 2 + l \times w \times 3 \\ = wh + 3lw$$



Cylinder (circular prism)

Volume = area of end region x height

$$V = \pi r^2 \times h = \pi r^2 h$$

Surface area = (2 x area of base) + (Circumference x height)

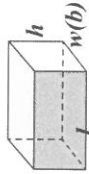
$$SA = 2\pi r^2 + 2\pi r h \\ = 2\pi r(r + h)$$

All prisms    Volume = Area of end region x length (or area of base x height)  
SA = sum of areas of each side

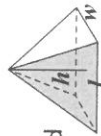
## Introduction

Review the names and properties of the 3D solids with the students.

Pyramids and prisms are named by the base. Prisms have two bases that are parallel to each other, whereas pyramids meet at a point, which is called an apex (a special vertex).



Rectangular prism



Rectangular pyramid

**Nets** - The nets show the flat 2D shape that may be folded to make the 3D object. Often there is more than one net that may be folded to form a 3D object, however, the nets that are supplied are ideal for focusing students on the component faces of a 3D solid and will help them visualise how the surface area may be calculated. The nets for associated shapes are the same colour. The nets for the cone and the cylinder are both orange.

**Euler's law** - states that the number of vertices plus the number of faces in every 3D solid will always equal the number of edges plus two, expressed as  $V + F = E + 2$ .

- Vertices (V) - points where three or more edges meet
- Faces (F) - a flat surface
- Edges (E) - the straight or curved line segment formed when two surfaces are joined together



Square pyramid  
 $V=5$   $F=5$   $E=8$

There is often some confusion when it comes to surfaces of cylinders, cones and spheres. A surface may be curved or flat. When a surface is flat we call it a face. A cylinder has two flat surfaces (faces) and one curved surface.



Cone

$$V=1$$
  $F=1$   $E=0$



Cylinder

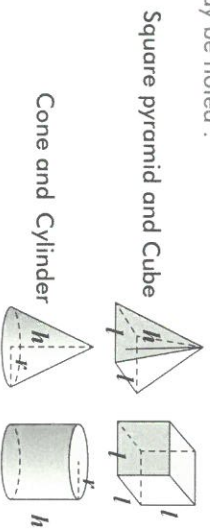
$$V=0$$
  $F=2$   $E=0$

## Introducing the 3D solids

Ask some students to sort the pieces into groups. Likely they will sort them into pyramids and prisms. Discuss the general classification of pyramids and prisms. Note the cylinder is a circular prism – a special type of prism. The cone is a circular pyramid – a special type of pyramid.

### Showing other relationships

When the following 3D shapes are placed next to each other, further relationships may be noted :



Notice how each of these 3D shapes is the same height.

## Volume and Capacity

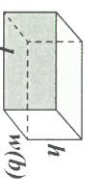
Volume - amount of space that a 3D object takes up, generally measured in cubic centimetres (cc) or cubic metres (m<sup>3</sup>).

Capacity - how much a container may hold and is generally measured in millilitres (ml) or Litres (l).

Students may compare how much the prism and the associated pyramid hold (ie the pyramid that has the same base and the same height as the prism). Pouring water or a dry mix such as rice from the pyramid into the associated prism will show that a pyramid holds one-third the amount of associated prism. That is why the formula for the volume of a pyramid is one-third the volume of the associated prism. Note that the thin faces of the plastic models means that the volume and capacity of these models will be almost the same.

For example, the volume of a cube is  $l \times l \times l$  and the volume of the associated square pyramid is  $1/3 \times l \times l \times l$ .

Prisms - Volume of a prism is calculated by determining the area of the base (or end) and multiplying by the length. For example :



$$\text{Volume of a rectangular prism} = l \times w \times h$$

## Surface Area

When you wrap a present you are basically covering the surfaces of a 3D object. When calculating the surface area of a 3D object students need to calculate the area of all the individual faces. In the case of a cube this is relatively easy because all the faces are the same, so once you have calculated the area of one square ( $l \times l$ ) all you need to do is multiply by six to determine the surface area of the cube.

Visualising the component parts that make up a 3D shape is much easier with the net to act as a guide. Students will be able to literally 'see' that a pentagonal pyramid is made up of a pentagon at the base and five triangles that are all the same size (congruent).

### Pentagonal pyramid



**Nets** - the flat 2D shape that may be folded to make a 3D shape. The net provided for the cube is a standard 'r' shape. Set the challenge for students to find other nets that fold to make a cube. It may surprise the students that there are 11 different nets that may be folded to make a cube.